

Simplifying Rational Expressions Notes

Rational Expression

A rational expression or simply fraction is a quotient of two algebraic expressions or polynomials.

Two Types of Fraction

1. Proper fraction is one whose degree of polynomials in the numerator is less than the degree of the polynomials in the denominator.

2. Improper fraction is one whose degree of the polynomials in the numerator is greater than or equals the degree of the polynomial in the denominator.

A fraction is said to be in simplified form if the numerator and the denominator have no common factor except ± 1 . If the common factor appears in the numerator and denominator, such as can be removed by division using property of a set or real numbers

$$\frac{ac}{bc} = \frac{a}{b}; b \neq 0, c \neq 0$$

$$\frac{x-y}{x-y} = 1$$

and

$$\frac{x-y}{y-x} = -1$$

This means that the value of a fraction is unchanged if its numerator and denominator are both multiplied by the same quantity or divided by the same quantity, provided that quantity is not zero.

Sample Problem 1: Simplify the following rational expressions.

1. $\frac{x^2-4}{2x-4}$

Solution:

$$\frac{x^2-4}{2x-4} = \frac{(x+2)(\cancel{x-2})}{2(\cancel{x-2})} = \frac{x+2}{2}$$

2. $\frac{2x+4y}{x^2+2xy}$

Solution:

$$\frac{2x+4y}{x^2+2xy} = \frac{2(\cancel{x+2y})}{x(\cancel{x+2y})} = \frac{2}{x}$$

3. $\frac{2x^2-3x+1}{2x^2+x-1}$

Solution:

$$\frac{2x^2-3x+1}{2x^2+x-1} = \frac{(\cancel{2x-1})(x-1)}{(\cancel{2x-1})(x+1)} = \frac{x-1}{x+1}$$

4. $\frac{bx-by-4x+4y}{b^3-16b}$

Solution:

$$\frac{bx-by-4x+4y}{b^3-16b} = \frac{b(x-y)-4(x-y)}{b(b^2-4)}$$

$$\frac{(x-y)(\cancel{b-4})}{b(b+4)(\cancel{b-4})} = \frac{x-y}{b(b+4)}$$

5. $\frac{r^3-ar+2ar^2-2a}{r^3-ar}$

Solution:

$$\frac{r^3-ar+2ar^2-2a}{r^3-ar} = \frac{r(r^2-a)+2a(r^2-a)}{r(r^2-a)}$$

$$\frac{(r+2a)(\cancel{r^2-a})}{r(\cancel{r^2-a})} = \frac{r+2a}{r}$$